6.7 Solving Logarithmic Equations

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Exponential Equations:

$5^{2x-4} = 25$ vs. $5^{2x-4} = 18$

$5^{2x-4} = 5^2$ $\qquad$ $2x-4 = \log_5 18$

$2x-4 = 2$ $\qquad$ $2x-4 = \log_5(8)$

$\boxed{x=3}$

Logarithmic Equations:

$\log_2(x-8) = \log_2(2x+1)$ vs. $\log_2(x-8) = 3$

$x-8 = 2x+1$ $\qquad$ 2^3

$\boxed{\cancel{-9 = x}}$ $\varnothing$ extraneous solution $\qquad$ $x-8 = 8$, $+8$ $+8$

$\cancel{\log_2(-17) = \log_2(-17)}$ $\qquad$ $\boxed{x=16}$

$\varnothing \neq \varnothing$

Steps:

1. Manipulate all logs to same side
2. Condense all logs to a single log
3. Isolate the log
4. Apply an exponential to both sides
5. Solve for x
6. Check for extraneous solutions

ex

$\ln(x-2)+3 = 6$

(3) $-3 \quad -3$

$\ln(x-2) = 3$

(4) $e^{\ln(x-2)} = e^3$

$x-2 = e^3$

(5) $x-2 = 20.09$

$+2 \quad +2$

$\boxed{x = 22.09}$

(6) $\ln(22.09-2)+3 \overset{?}{=} 6$ ✓

ex

$\log(5x) = 2 - \log(x-1)$

(1) $+\log(x-1) \quad +\log(x-1)$

$\underbrace{\log(5x)+\log(x-1)}_{\text{condense}} = 2$

(2) $\log(5x\cdot(x-1)) = 2$

(4) $10^{\log(5x\cdot(x-1))} = 10^2$

$5x(x-1) = 10^2$

(5) $5x^2 - 5x = 100$

$-100 \quad -100$

$5x^2-5x-100 = ☺$

$5(x^2 - x + 20) = 0$ (A, M)

$5(x-5)(x+4) = 0$

$\boxed{x=5}$ ✓ $\qquad$ $\boxed{\cancel{x=-4}}$ extraneous solution

(6) $\log(5\cdot5) \overset{?}{=} 2-\log(5-1)$

$\log(5\cdot{-4}) \overset{?}{=} 2-\log(-4-1)$

cannot have a negative #

cannot have a negative #
inside a log